Droplet Traffic at a Simple Junction at Low Capillary Numbers

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We report that, when a train of confined droplets flowing through a channel reaches a junction, the droplets either are alternately distributed between the different outlets or all collect into the shortest one. We argue that this behavior is due to the hydrodynamic feedback of droplets in the different outlets on the selection process occurring at the junction. A “mean field” model, yielding semiquantitative results, offers a first guide to predict droplet traffic in branched networks.

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Two phase capillary flows are ubiquitous in nature as well as in many man-made systems. Of particular interest are flows in networks, of liquids containing deformable dispersions of size comparable to the capillary size. Illustrative examples include blood circulation, flows in porous media, and droplet microfluidics, where droplets of one phase are formed and transported in small channels by a moving immiscible carrier phase [1–6]. Potential applications of droplet microfluidics encompass chemical or biochemical analysis, studies of chemical kinetics, high throughput screening and combinatorial chemistry, and the fabrication of tailored microparticles [1,6]. Essential to these tasks is the possibility of generating many monodisperse individual microreactors (droplets) that do not disperse and lose their integrity while flowing through the system [5]. However, to achieve the parallelization desirable for high throughput, it is also necessary to control the traffic of droplets in the microsystem, as well as tasks such as stirring, dividing, recombining, storing, etc. [1,5–7].

In this Letter, we consider a basic yet essential element of droplet traffic control, which is the effect of a single “T junction” where the flow is separated into two streams. We focus on junctions of simple geometry, with no neck or feature hindering droplet motion, and consider regimes where the viscous stresses are small so that the droplets do not appreciably deform and, consequently, do not break [7]. As sketched in Fig. 1, the question is then to understand how the droplets and carrier phase divide between the two branches. We start by reporting experiments which show that, depending on their dilution, the droplets are either sorted exclusively into one branch or partitioned between both. We then argue that this behavior is due to the hydrodynamic feedback of drops in the two outlets on the selection process occurring at the junction. The outcome is a complex dynamical process, but a simple “mean field” model is shown to yield a semiquantitatively predictive picture.

Experiments.—Our experiments are performed with a “millifluidic” device, in which cylindrical glass capillaries of internal radius \( r_c = 650 \ \mu \text{m} \) are assembled using homemade Plexiglas connectors. Monodisperse droplets are formed using a calibrated cylindrical needle of diameter 510 \( \mu \text{m} \) (Harvard Inc.) centered with respect to a capillary tube (see Fig. 1). Using separate syringe pumps (Harvard PHD 2000), water (millipore water, viscosity \( \eta_w = 1 \text{ mPa} \cdot \text{s at } T = 20 \degree \text{C} \)), and an oil (either silicone oil (Fluka, France), viscosity \( \eta_o = 100 \text{ mPa} \cdot \text{s at } T = 20 \degree \text{C} \), or sunflower oil (Leader Price, France), viscosity \( \eta_o = 50 \text{ mPa} \cdot \text{s at } T = 20 \degree \text{C} \)) are infused, respectively, through and around the central needle, as depicted in Fig. 1. The oil \( Q_o \) and water \( Q_w \) flow rates are independently controlled and adjusted in order to form almost monodisperse water droplets that are emitted at a constant rate \( f \). Their radius \( r \), as deduced from their volume \( \frac{Q_w}{f} \), is always smaller than the internal radius \( r_c \) of the capillary so that the droplets remain spherical.

FIG. 1. Sketch of the experimental setup.
An additional injection of oil downstream at a constant flow rate $Q_d^o$ dilutes the system and increases the distance $\lambda$ between two successive droplets while keeping their size unchanged. The channel is thus filled with a periodic “train” of droplets moving at constant velocity $U = Af$, which is directed towards a simple symmetric T junction. Both sidearms of the junction have the same constant cross section as the initial channel, $S = \pi r_c^2$, but different lengths $L_1$ and $L_2 > L_1$, with $R = L_2/L_1$ a nondimensional measure of their asymmetry. Their ends are connected to the same atmospheric pressure $P_0$. Thus, our setup is somewhat analogous to the microfluidic circuit used by Link et al. [7] to study the fragmentation of droplets at a junction. However, we prevent here such a fragmentation process by working with small enough droplets and flow rates (capillary numbers in our experiments are $\sim 10^{-2}$ and Reynolds number $\sim 10^{-1}$). For each of the three arms of the junction (the inlet and the two outlets), images of the flows are captured and recorded with a CCD video camera (Hamamatsu digital camera C4742-95, Japan), and the droplet frequency in each arm is measured using a helium neon laser beam and a photodiode connected to a digital oscilloscope (Hewlett Packard). This gives us access to the average distances between the droplets $\lambda_1$ and $\lambda_2$ and to their passage frequencies $f_1$ and $f_2$ in the two outlets.

In an experiment, we vary progressively the dilution flux, at fixed frequency $f$ and drop size, measured by the nondimensional ratio $q = Q_o/Q_w$, where $Q_o = Q_d^o + Q_d^o$ is the total oil flow rate. Figure 2 shows the corresponding evolution of the droplet pattern at the junction. For small values of $q$, or low dilutions, droplets are observed in both outlets of the junction, essentially regularly spaced. Upon increase of dilution, i.e., for larger values of $q$, the fraction of droplets going into the longest sidearm decreases. Furthermore, the distribution of droplets in each outlet appears less regular and defects or “holes” are observed. As a critical threshold value $q_c$ is approached, the distance between droplets in the longest arm rises rapidly, and above that threshold all droplets go into the shortest arm. The junction acts then as a perfect “hydrodynamic” filter, despite the absence of a pore or obstacle of size smaller than that of the droplets.

A first qualitative picture.—To gain some insight into the mechanisms at work in this transition, we consider first a very dilute system. The droplets in our system are essentially centered in the inlet capillary, so that we anticipate that each single droplet arriving at the locally symmetric junction follows the dominant stream and enters the channel with the highest flow rate. At high dilutions, this is equivalent to systematically going through the shortest arm, which has a lower hydrodynamic resistance. Decreasing dilution may change things, since this results in more droplets in the shortest sidearm, which raises its hydrodynamic resistance, possibly up to the point where the flow rate is larger into the longest arm. Obviously, this leads to a complex dynamical process whereby a droplet arriving at the junction reacts to the instantaneous state of the system to “decide” in which arm to go. The choice modifies the balance of the system, so the next droplet faces a different situation and can either follow its predecessor or make the opposite “choice.” A complete description of this problem is a complex task that we will not attempt here. Note that hydrodynamics results in the droplet arriving at the junction being influenced by (i) the details of the flow in the neighborhood of the junction, i.e., at distances of order a few capillary diameters, which can significantly be modified by the presence of other droplets there and (ii) global quantities such as the instantaneous flow rates in the two arms. At low densities, the interactions between droplets are thus fully described by the increase of hydrodynamic resistance generated by the droplets which have already gone through the junction but have not yet exited the system, whereas at higher densities more complex interactions between consecutive droplets are expected [point (i) above]. We elaborate below a description of the former case and quantify first the increase with droplet concentration of the hydrodynamic resistance of a channel.

A train of drops in a simple channel.—Consider a train of $n$ droplets in a capillary of section $S = \pi r_c^2$ and length $L = n\lambda$. At low Reynolds and capillary numbers, linear relations hold between the various kinematic quantities (we are far from the Bretherton regime of large droplets “rubbing” against the walls with a friction that depends on the capillary number). The total flow rate $Q$ and the velocity $U$ of the droplets are related by:

$$U = \beta \frac{Q_w + Q_o}{S} = \beta \frac{Q}{S}$$

with a system dependent dimensionless coefficient $\beta$. In a dilute situation, the pressure drop across the channel is the sum of pressure drops over droplet-free and droplet-containing sections, so that one can write:

$$\Delta P = Q \frac{\eta_o L}{8 \pi r_c^4} \left(1 + \frac{L_d}{L}\right) = Q \frac{\eta_o L}{8 \pi r_c^4} \left(1 + \frac{L_d}{\lambda}\right)$$

where the length $L_d$ measures the additional resistance per
For droplets of radius \( r = (3\Omega_d/4\pi)^{1/3} < r_c \), \( \beta \) and \( L_d/r \) are nondimensional functions of the viscosity ratio \( \eta_o/\eta_w \) and of the geometrical factor \( r/r_c \) (corrections should arise for larger droplet concentration \( n/L \) and capillary number).

The above formulas are supported by experiments performed in a capillary tube of section similar to those in Fig. 1, of length \( L = 128 \text{ mm} \), for various droplet volumes and total flow rates \( Q \). The droplets are produced at a rate \( f \) as previously described. Their velocity is deduced from their distance \( U = \lambda f \), and the pressure drop \( \Delta P \) measured using side channels. \( U \) is found to vary linearly with \( Q \) (Fig. 3), with values for \( \beta \) ranging from 2 to values close to 1 as the droplet size is increased. The variation of \( \Delta P/Q \) with droplet concentration \( n/L = \lambda^{-1} \) (Fig. 3) shows that small droplets hardly alter the hydrodynamic resistance of the channel, whereas larger ones significantly increase it, with values of \( L_d \) in the millimeter range.

A first quantitative model for the \( T \) junction.—We now return to the droplet traffic at a \( T \) junction. We assume that the arms are very long (compared to the droplet size and \( L_d \)), so that the relative fluctuations in time of the hydrodynamic resistances are weak, and attempt a steady-state description using average quantities.

From our earlier discussion, we expect the “filter” regime to prevail as long as the hydrodynamic resistance of the short arm, with the droplets it contains, is smaller than that of the droplet-free long arm. Assuming a steady (average) number \( n_1 \) of droplets in the short arm of length \( L_1 \), we write mass conservation of both oil and water at the junction and partition of the total flow rate according to the ratio of the hydrodynamic resistances as described by Eq. (2): \( Q_2/Q_1 = (L_1 + n_1 L_d)/L_2 \).

Then the frequency and concentration of the droplets in the short arm are related to the ratio of oil to water in the incoming channel \( q = Q_o/Q_w \) by:

\[
    f_1 = f; \quad \frac{n_1}{L_1} = \frac{1}{\lambda_1} = \frac{1}{L_d} \left( R (q + 1) - \gamma \right),
\]

with \( \gamma = S L_d/(\beta \Omega_d) = (3/4)(r_c/r)^2(L_d/r)^2 \) a nondimensional “local” parameter characteristic of the droplet in channel geometry and viscosity contrast, and \( R = L_2/L_1 \) the “global” length asymmetry between the two arms. The requirement of lower resistance of the short arm is a requirement of high dilution:

\[
    q \geq \dot{q}_c = \frac{2\gamma - R + 1}{R - 1}.
\]

Note that, for very asymmetric junctions, \( R \approx 2\gamma + 1 \), only the filter regime should show up.

The description of the higher density regime where droplets are present in both channels is much more complex. We nevertheless proceed with a steady-state description. Taking steady numbers of droplets \( n_1 \) and \( n_2 \) in the two branches, we write conservation equations for both phases and make use of the relations (1) and (2). To close the system, an additional relation is needed, which mimics in this steady-state picture the average outcome of the dynamical selection rule for each individual droplet arriving at the junction. In lack of a prescription to write such a relation, we put forward two possible choices. The first choice builds on the low density picture whereby a droplet chooses the outlet with the largest flow rate. Then, if droplets alternatively go into the two arms, the corresponding net flow rates should remain close in the limit of weak relative fluctuations in the number of droplets. This leads to an “equality rule” for the total flow rates \( Q_1 \approx Q_2 \), which leads to simple formulas for the passage frequencies in the two arms (\( q \leq \dot{q}_c \)):

\[
    f_1 = f \left( \frac{1}{R + 1} \left( R + \frac{1 + q}{1 + \dot{q}_c} \right) \right); \quad f_2 = f \left( \frac{1}{R + 1} \left( \frac{\dot{q}_c - q}{1 + \dot{q}_c} \right) \right).
\]

The second choice elaborates on an empirical observation. We collect the liquid exiting through each arm and wait for droplet coalescence that leads to macroscopic phase separation. For the lowest values of \( q \), we observe that the total flow rates in the two arms differ but that the oil rates are almost identical. Substituting the corresponding empirical rule \( Q_{o1} = Q_{o2} \) in our system of equations, we arrive at different predictions:

\[
    f_1 = f \left( \frac{R + q/\dot{q}_c}{R + 1} \right); \quad f_2 = f \left( \frac{1 - q/\dot{q}_c}{R + 1} \right).
\]
The value of \( q_c = \frac{2(\gamma^2 R^2)}{1+R^2} \). For both choices, formulas for the average distances between droplets \( \lambda_1 \) and \( \lambda_2 \) are similarly obtained.

To test the above description, we report in Fig. 4 a typical set of data, corresponding to water droplets in silicone oil (volume \( \Omega_d = 724 \) nl). The frequency ratios \( f_1/f \) and \( f_2/f \) and distances between drops \( \lambda_1 \) and \( \lambda_2 \) are plotted as a function of \( q \), together with the predictions above. The value of \( \gamma \) is borrowed from the single tube experiments reported above so that there is no free parameter. The behavior far from the transition is well captured by Eq. (3) in the filter regime and Eqs. (6) for the dense regime. The location of the transition is predicted with a 20% accuracy by (4), but the behavior close to the transition is not captured by our simple analysis. However, the two branches corresponding to Eqs. (5) and (6) provide a useful guide. Similar agreement is reached with other droplet sizes (ranging from 150 to 900 nl) and with sunflower oil as carrier fluid. Altogether, given its simplicity and the absence of free parameters, we find our model satisfactory in semiquantitatively predicting the observed behavior.

We are currently studying the dynamics in the vicinity of the transition, to characterize and model the temporal and spatial fluctuations observed in the experiments. Finite size effects may have to be taken into account, as well as interactions between consecutive droplet. Note that, for weaker asymmetry (\( R \) closer to 1), the transition should occur at lower concentrations, possibly in conditions where such interactions are negligible and do not interfere with the transition. On the other hand, these interactions may be the reason why Eqs. (6) capture the observed behavior at low dilution, with the continuous phase equally distributed in this regime.

**Concluding remarks.**—Many features of our analysis for this specific T junction can be applied to other geometries and topologies, especially those of microfluidic devices, with values of \( \beta \) and \( L_d/r \) in the friction laws (1) and (2) adapted to the specific local geometry (square channels, droplets confined by the walls, etc.). In particular, at low droplet concentrations, a simple strategy is to compute the flow rates given the instantaneous droplet distribution and then decide at each junction the fate of an incoming droplet by comparing the corresponding flow rates (possibly taking into account the specific local geometry of the junction, as well as the possibility of breakup at higher capillary numbers).

In summary, we have studied the behavior of nonfragmenting droplets at a T junction. Depending on the distance between the incoming droplets, the T junction acts as a hydrodynamic sieve or as a partial sorter. Beyond a threshold dilution factor, the droplets are all driven towards the channel of lowest hydraulic resistance. At higher density, the droplets alter significantly the resistance of the two arms and are sorted in the two outlets along a complex dynamical scheme. We have provided bounds for the net outcome of this sorting process. Our results offer a first guide for the design and modeling of the traffic of droplets in complex branched networks, a necessary step towards parallelized “droplet-based-lab-on-a-chip” devices.

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**References**


